Baseline Time–Series Momentum and Grid–Sized Enhancement

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Abstract

This report studies a managed futures trend-following strategy built on time-series momentum (TSMOM) signals across 58 liquid futures markets from 1969 to 2025. Following the literature on TSMOM, beginning with Moskowitz, Ooi, and Pedersen [Moskowitz et al., 2012], we construct a baseline long–short trend rule using volatility-scaled returns and evaluate it under three volatility estimation schemes: a full-sample constant volatility, an expanding-window estimator, and an exponentially weighted moving average (EWMA). We then introduce an enhancement that augments the baseline with a GRID mechanism, which confirms directional signals using raw-return trends and scales exposure discretely with trend strength. A large-scale grid search is conducted to identify the best in-sample configuration under realistic trading and rolling costs. In-sample (1969–2014), the GRID overlay improves net returns and Sharpe ratios relative to the standard TSMOM benchmark. Out-of-sample (2015–2025), the best in-sample specification continues to deliver positive returns, though with a materially lower Sharpe ratio. The results highlight both the robustness and the limitations of strength-based sizing in trend-following strategies.

Contents

1 Introduction

Trend-following strategies in futures markets have a long empirical history and form the foundation of many managed-futures and CTA portfolios. A central and influential implementation is time-series momentum (TSMOM), which takes long positions in assets with positive past returns and short positions in those with negative past returns. This phenomenon has been documented across equities, bonds, commodities, and currencies, most notably by [Moskowitz et al., 2012], and continues to serve as a benchmark in both academic research and quantitative investment practice.

The objective of this project is to reproduce, analyze, and extend a volatility-targeted TSMOM strategy using a broad panel of 58 futures contracts from 1969 to 2025. We begin by constructing a baseline trend-following rule in which each asset's position is determined by the sign of its past twelve months of volatility-scaled returns. To better reflect real-world implementation, we compare three volatility estimators: a static full-sample estimate, a recursive expanding-window estimate, and an exponentially weighted moving average (EWMA) model. These variations allow us to study how different assumptions about risk modeling affect the realized volatility, turnover, and performance of a standard TSMOM portfolio.

We then introduce a trend-strength overlay—the GRID mechanism—to address a limitation of binary trend signals: all positive trends lead to the same exposure regardless of their magnitude or persistence. The GRID framework uses raw-return trends to confirm the TSMOM direction and scales exposure in discrete steps as trend strength increases. A high-dimensional grid search explores thousands of parameter combinations, selecting the configuration that maximizes net Sharpe ratio over the in-sample period (1969–2014), accounting for realistic trading and rolling costs.

Finally, the best in-sample configuration is evaluated out-of-sample from 2015 to 2025 to assess robustness. While the GRID-sized strategy continues to deliver positive net performance, its Sharpe ratio declines relative to the in-sample results, reflecting shifts in market volatility, trend persistence, and the binding of the gross-exposure cap. The overall findings illustrate how strength-based sizing can enhance classical trend-following while also underscoring the challenges of maintaining performance in changing market regimes.

2 Data

2.1 Data Description

The dataset consists of monthly total returns spanning 1969-01-31 to 2025-05-30. The main sample used for estimation extends from 1969-01-31 through 2014-12-31, producing 552 monthly observations across 58 assets. The asset universe contains 26 commodity indices (COMM), 14 equity indices (EQ), 10 fixed-income indices (FI), and 8 foreign exchange series (FX). All series are aligned on a monthly frequency.

Assets are included as soon as their individual histories begin. Because each series represents a broad index rather than a single tradable security, we allow assets to enter dynamically at their inception rather than imposing a common starting date. This preserves the informational richness of the cross-section and reflects realistic availability of tradable instruments over time.

For the baseline strategy and the computation of summary statistics, the full history is used, with assets included as soon as their return streams become available. As these assets represent broad indices rather than individual securities, we allow each index to enter the sample at its inception.

For the expanding-window strategy, the information set at each time is restricted to returns available on or before that month, ensuring no look-ahead bias. The same principle applies to the exponentially weighted moving average (EWMA) estimator, which differs only in its weighting scheme. Both methods require an initial estimation window, leading to a slightly later strategy start date.

To evaluate model robustness, the full dataset from 1969-02-28 to 2025-05-30 is partitioned into:

- In-Sample (IS): 1969-02-28 to 2014-12-31

  This window is used to estimate model parameters and determine the "best" configuration of the strategy (e.g., signal decay rate, volatility target, risk estimator speed).

- Out-of-Sample (OOS): 2015-01-01 to 2025-05-30

  The best IS configuration is frozen at the end of 2014 and then applied mechanically throughout the OOS period. Because the data are monthly, the first OOS return appears on 2015-01-30 and the final observation on 2025-05-30.

This setup mirrors industry standards by ensuring that parameter selection is performed only within the IS sample and that OOS performance reflects true forward application without further recalibration.

2.2 Data Summary

=== Per Asset Class (annualized) ===

Asset Class	Avg	Std	SR
COMM	5.20%	14.25%	0.37
EQ	6.64%	15.94%	0.42
FI	2.69%	8.28%	0.32
FX	1.05%	7.08%	0.15

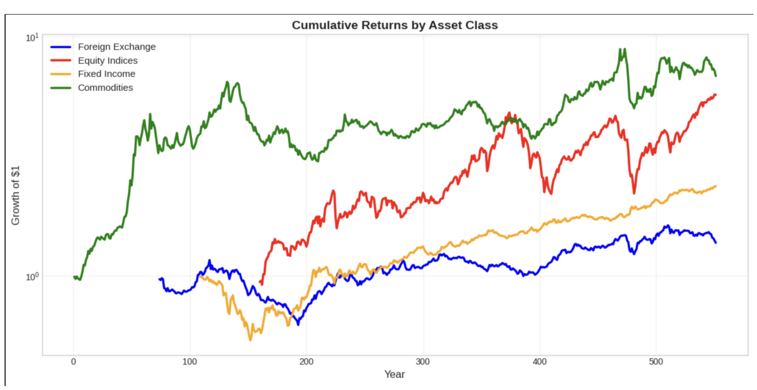

Figure 1: Asset Category Cumulative Return

=== Per Instrument (annualized and sorted by Sharpe ratio) ===

Instrument	Avg	Std	SR
TU	1.57%	1.67%	0.94
CB	4.67%	5.45%	0.86
UB	2.49%	3.30%	0.75
FB	3.12%	4.16%	0.75
EC	1.09%	1.46%	0.75
DT	4.01%	5.49%	0.73
TY	4.59%	6.82%	0.67
UZ	0.90%	1.36%	0.66
MD	9.37%	16.75%	0.56
SP	7.85%	15.18%	0.52
MP	5.48%	10.68%	0.51
GS	3.54%	6.91%	0.51
YM	6.18%	14.13%	0.44
RL	8.04%	19.27%	0.42
ND	11.31%	27.11%	0.42
US	4.43%	11.25%	0.39
ZB	13.03%	34.33%	0.38
SS	0.38%	1.08%	0.35
DA	9.65%	28.98%	0.33
ZM	10.45%	31.85%	0.33
AN	3.74%	11.59%	0.32
HS	8.23%	25.51%	0.32
ZD	4.84%	15.16%	0.32
ZA	10.05%	31.48%	0.32
ZK	7.86%	25.95%	0.30
ZF	4.39%	14.70%	0.30
AP	3.93%	14.89%	0.26
AX	5.70%	22.24%	0.26
ZT	4.47%	17.54%	0.25
LX	3.59%	14.61%	0.25
ZU	8.43%	34.42%	0.24
ZL	7.64%	31.45%	0.24
ZZ	6.50%	27.00%	0.24
ZP	6.10%	27.30%	0.22
XX	2.92%	15.51%	0.19
MW	4.62%	25.04%	0.18
ZS	5.20%	28.20%	0.18
JO	5.28%	31.77%	0.17
BN	1.59%	10.50%	0.15
KC	5.67%	37.88%	0.15
CA	2.63%	18.46%	0.14
CT	3.36%	26.75%	0.13
ZI	4.09%	33.37%	0.12
CN	0.75%	6.85%	0.11

=== Per Instrument (annualized and sorted by Sharpe ratio) ===

Instrument	Avg	Std	SR
XU	1.69%	19.23%	0.09
ZG	1.35%	19.43%	0.07
SN	0.46%	12.30%	0.04
NK	0.37%	21.42%	0.02
FN	0.17%	11.17%	0.02
ZO	-0.41%	32.36%	-0.01
ZW	-0.56%	27.14%	-0.02
JN	-0.42%	11.89%	-0.04
ZC	-1.32%	26.41%	-0.05
SB	-2.95%	38.76%	-0.08
CC	-2.64%	29.14%	-0.09
ZR	-4.18%	28.58%	-0.15
ZN	-9.09%	51.25%	-0.18
LB	-6.64%	30.09%	-0.22

3 Strategy

3.1 Baseline

We used a simple **long–short** rule that we rebalance each month. Let $r_{i,t}$ be the monthly return of instrument i in month t. First, we scale past returns to a 40% annualized volatility using the full–sample realized volatility of each instrument. Let s_i be the full–sample standard deviation of monthly returns and let $v_i = s_i\sqrt{12}$ be its annualized value. The scaled return is

$$\tilde{r}_{i,t} \;=\; r_{i,t}\,\frac{0.40}{v_i}.$$

Next, we form a 12–month **momentum signal** from the scaled series. For simplicity, we sum the last twelve scaled monthly returns:

$$m_{i,t} \;=\; \sum_{k=1}^{12} \tilde{r}_{i,t-k}.$$

The trading position is the sign of the signal. If the signal is positive we take a long position $p_{i,t} = +1$. If the signal is negative we take a short position $p_{i,t} = -1$:

$$p_{i,t} \;=\; \operatorname{sign}(m_{i,t}) \in \{-1, +1\}.$$

These steps are repeated every month: scale using the same full–sample volatility, compute the past 12–month signal, set the position by its sign, and hold the position for one month until the next rebalance.

3.2 Time varying volatility calculation: Expanding window and EWMA

Let $r_{i,t}$ denote the monthly return of instrument i at month t. We seek a time-varying estimator $\sigma_{s,t}$ for annualized volatility to use in risk scaling.

3.2.1 Expanding-Window Volatility (Equal Weights)

The expanding estimator uses *all* past data up to $t-1$ with equal weight:

$$\bar{r}_{i,t-1} = \frac{1}{N_{i,t-1}} \sum_{u=1}^{t-1} r_{i,u},$$
$$\hat{\sigma}_{i,t} = \sqrt{\frac{1}{N_{i,t-1}-1} \sum_{u=1}^{t-1} \big(r_{i,u} - \bar{r}_{i,t-1}\big)^2} \times \sqrt{12},$$

where $N_{i,t-1}$ is the number of available (non-missing) monthly observations up to $t-1$. We require a minimum history $N_{i,t-1} \geq N_{\min}$ (e.g., $N_{\min} = 36$ months); otherwise $\hat{\sigma}_{s,t}$ is set to NaN and the instrument is excluded that month.

Properties. Long memory, very stable; slow to adapt to regime changes. Early in the sample, estimates are noisy until $N_{i,t-1}$ grows.

3.2.2 EWMA Volatility (Exponential Weights)

EWMA assigns geometrically decaying weights to the past, emphasizing recent data. Let $\lambda \in (0,1)$ be the decay factor. The variance recursion is

$$\big(\hat{\sigma}_{i,t}^{(\text{ewma})}\big)^2 \;=\; (1-\lambda)\, r_{i,t-1}^2 + \lambda \, \big(\hat{\sigma}_{i,t-1}^{(\text{ewma})}\big)^2, \qquad t \geq 2,$$

with an initialization (e.g., expanding std after $N_{\min}$ months or the unconditional sample variance). The annualized estimator is $\hat{\sigma}_{i,t}^{(\text{ewma})} \times \sqrt{12}$.

Halflife parameterization. It is often clearer to choose a *halflife* h (in months) and set

$$\lambda \;=\; 2^{-1/h} \quad \Longleftrightarrow \quad h \;=\; \frac{\ln 2}{-\ln \lambda},$$

so that the weight on a shock decays to one half after h months. Shorter h (smaller memory) produces a faster, more reactive risk model.

Properties. Responsive to volatility clustering and regime shifts; smoothly "forgets" old data; requires selecting h (speed).

4 Strategy Results

4.1 Baseline

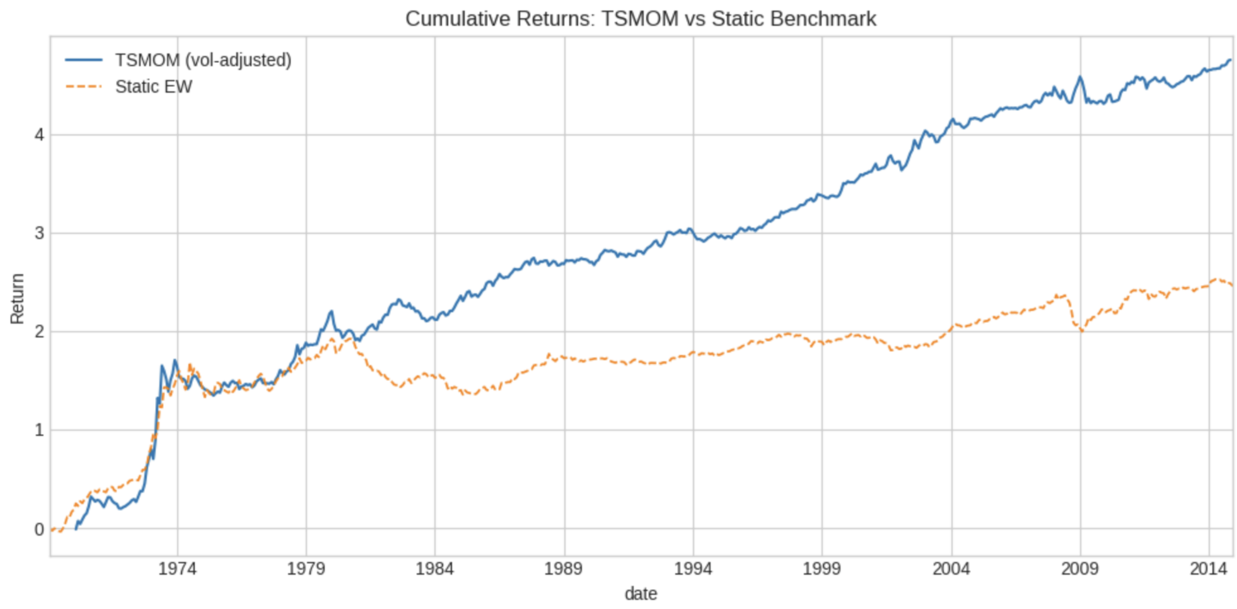

Figure 2: Baseline Return

Name	**Avg**	**Std**	**SR**	**Corr(Static)**
STATIC	5.34%	11.7%	0.45	nan
TSMOM	10.60%	15.41%	0.69	-0.01

Table 1: Baseline Strategy Results

4.2 Expanding window

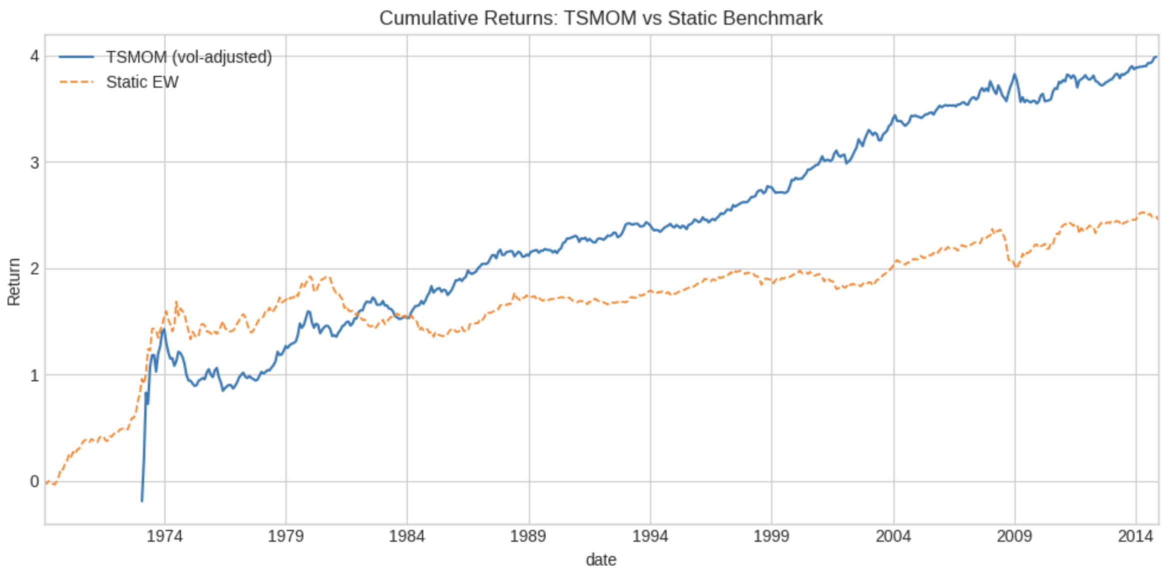

Figure 3: Expanding Window Return

Name	**Avg**	**Std**	**SR**	**Corr(Static)**
STATIC	5.34%	11.7%	0.45	nan
TSMOM	9.52%	17.66%	0.54	0.08

Table 2: Expanding Window Strategy Results

4.3 EWMA

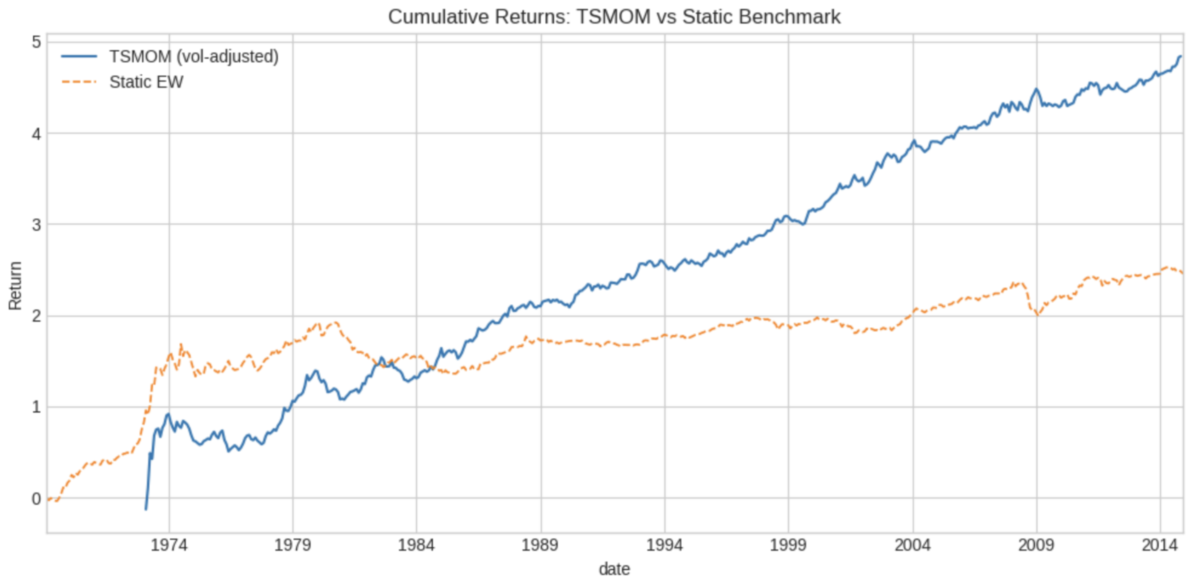

Figure 4: EWMA Return

Name	Avg	Std	SR	Corr(Static)
STATIC	5.34%	11.7%	0.45	nan
TSMOM	11.57%	15.09%	0.77	-0.04

Table 3: EWMA Strategy Results

The strategy's realized returns decline progressively as the volatility model becomes more realistic.

When moving to the expanding-window estimator, returns decrease as position sizing now depends only on past data, leading to conservative exposure early in the sample and during volatile periods. With the EWMA model, returns improves slightly, as the strategy force faster adjustments to recent volatility spikes.

5 Strategy Risk Characteristics

Table 4: Full-Sample Realized Annualized Volatility under Different Risk Models

Volatility Model	Description	Annualized Volatility (%)
Baseline (Full-Sample)	Constant σ_s from full historical data	15.40
Expanding Window	Equal-weighted historical volatility up to $t-1$	17.66
EWMA (Halflife = 12)	Exponentially weighted moving volatility	15.10

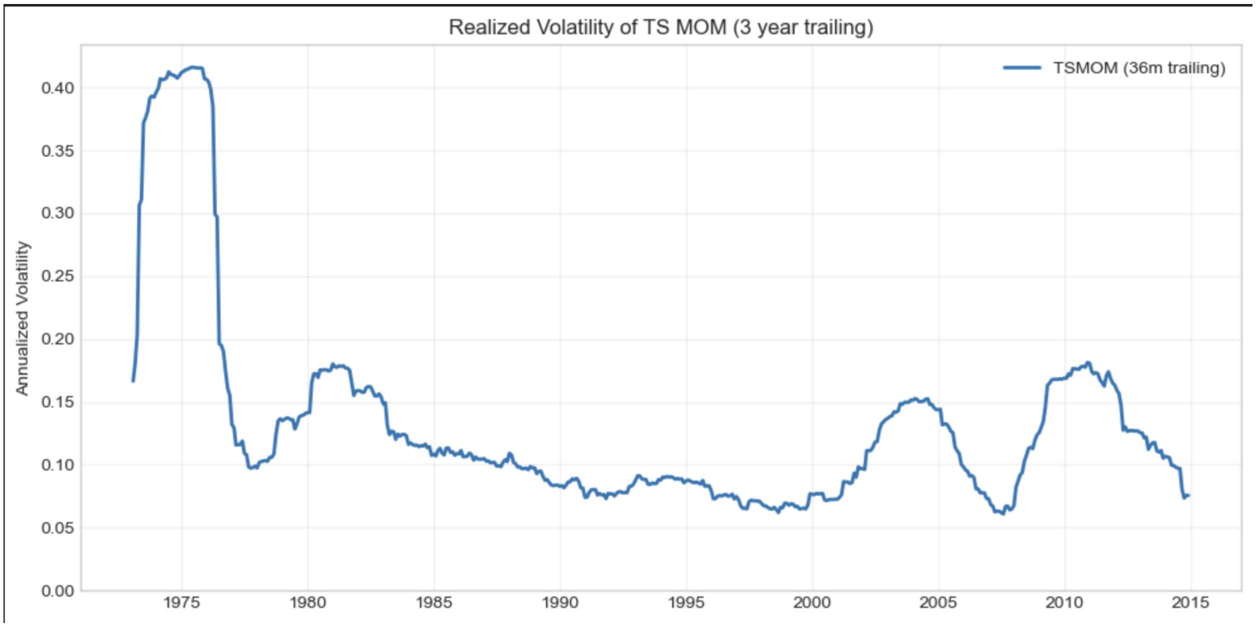

Figure 5: Realized Volatility of Baseline Strategy

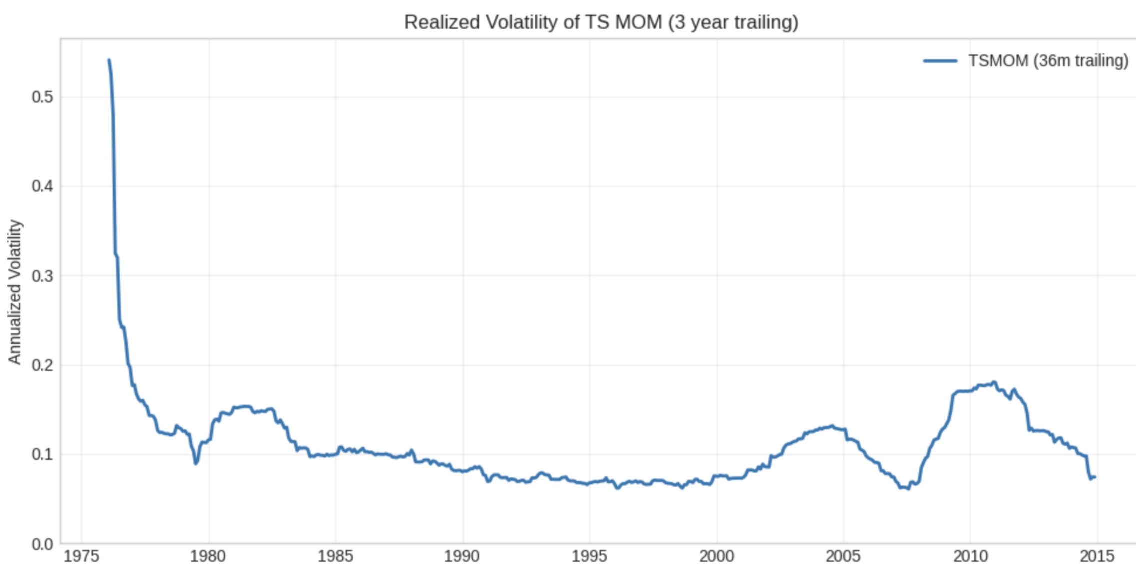

Figure 6: Realized Volatility of Expanding window

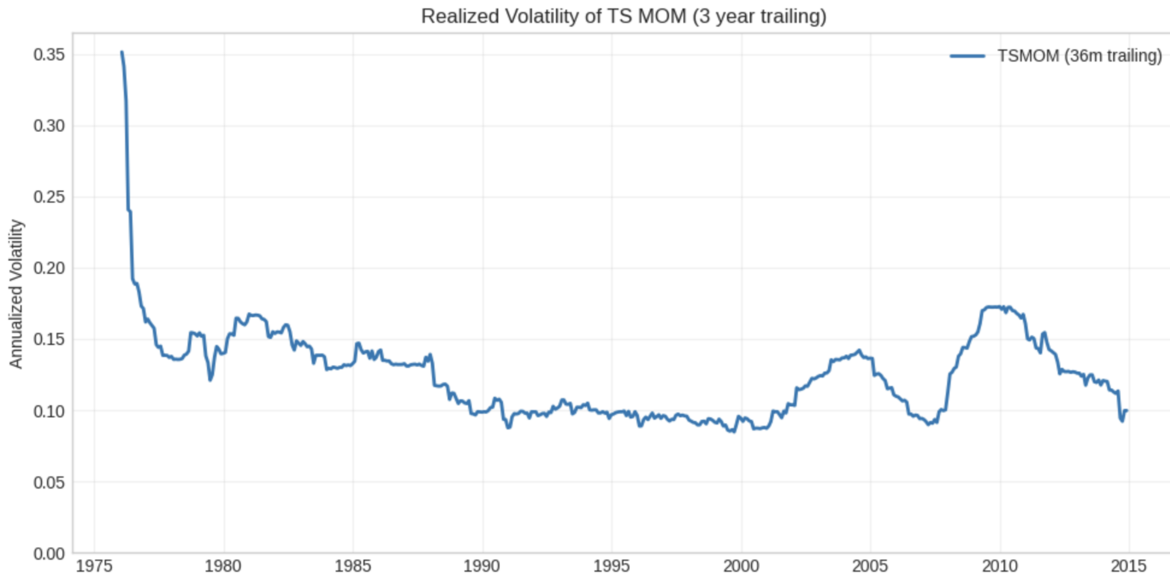

Figure 7: Realized Volatility of EWMA

The realized annualized volatility varies noticeably across the three risk-modeling approaches. The baseline model, which uses a full-sample constant volatility, produces 15.4%, reflecting its use of future information and complete historical smoothing. When switching to an expanding-window estimator, the realized volatility increases to 17.66%, as the model gradually incorporates new data and reacts to historical volatility clusters without the benefit of hindsight. The EWMA model (halflife = 12) brings the estimate back down to 15.1%, showing that exponential weighting smooths the long-run estimate while still adapting to recent market regimes.

Overall, volatility estimates become more realistic as we move from the static baseline to time-varying methods, with EWMA achieving a balance between responsiveness and stability.

6 Grid–Sized Trend Following Enhancement

6.1 Motivation

The baseline strategy is an equal weighted (EQ–Active) time series momentum (TSMOM) portfolio built on volatility targeted futures returns. Each instrument is scaled to a fixed annual volatility target of VOL_TARGET = 40% using an exponentially weighted moving average (EWMA) estimator and the portfolio then takes ± 1 positions based on the sign of past vol scaled returns with equal weight across all active instruments.

This binary signal treats all positive trends as identical regardless of their strength or persistence and likewise for negative trends. The GRID framework augments the baseline by introducing an additional trend indicator computed from raw unscaled returns a confirmation filter that requires agreement between the TSMOM and GRID directions and a discrete sizing rule that scales exposure as the GRID trend becomes stronger. The objective is to improve risk adjusted performance while respecting a cap on the total amount of volatility scaled risk that the portfolio can take.

6.2 GRID Signal and Portfolio Construction

The GRID component is built to sit on top of the existing volatility targeted TSMOM framework rather than to replace it. The construction proceeds in three layers. First we obtain a volatility scaled return series and a binary TSMOM direction. Second we compute a separate trend measure on raw unscaled returns and translate it into a discrete strength indicator. Third we combine the two signals through a confirmation rule and a strength based sizing rule that feeds into the final portfolio weights.

For each contract i we observe monthly raw returns $r_{i,t}$ and volatility scaled returns $\tilde{r}_{i,t}$. Volatility targeting uses an EWMA estimate $\hat{\sigma}_{i,t}$ and a fixed annual volatility target VOL_TARGET. The TSMOM direction is obtained from past $\tilde{r}_{i,t}$ over a signal lookback of sig_lookback months and takes values

$$s_{i,t} \in \{-1, 0, +1\}$$

depending on whether the recent vol scaled trend is negative flat or positive. The portfolio scale weights based on all available contracts with $s_{i,t} \neq 0$ at each date.

The GRID signal is constructed from raw returns in order to keep track of the underlying price move rather than the volatility scaled move. For a given grid_lookback we define a cumulative raw return

$$R_{i,t}^{\mathrm{GRID}} = \sum_{k=1}^{\mathrm{grid\_lookback}} r_{i,t-k}$$

which uses only information up to $t-1$. This cumulative trend is compared to three strictly increasing thresholds (ℓ_1, ℓ_2, ℓ_3). A positive trend that exceeds ℓ_1 counts as one GRID level a trend that exceeds ℓ_2 counts as at least two levels and so on. Symmetrically a negative trend that falls below $-\ell_1$ $-\ell_2$ or $-\ell_3$ triggers negative GRID levels. Let $k_{i,t}^{+}$ denote the number of positive levels and $k_{i,t}^{-}$ the number of negative levels. The GRID direction is

$$g_{i,t} = \mathbf{1}\{k_{i,t}^{+} > 0\} - \mathbf{1}\{k_{i,t}^{-} > 0\}$$

so $g_{i,t}$ equals plus one if the raw trend is strongly positive minus one if it is strongly negative and zero otherwise.

The first way GRID interacts with TSMOM is through a confirmation filter. Positions are held only when the binary TSMOM direction and the GRID direction agree. Formally the confirmation indicator is

$$c_{i,t} = \mathbf{1}\{s_{i,t} \neq 0 \text{ and } s_{i,t} = g_{i,t}\}$$

and contracts with $c_{i,t} = 0$ receive zero weight even if TSMOM alone would have taken a position. This step removes trades where the vol scaled trend and the raw trend tell conflicting stories.

The second way GRID enters is through discrete strength based sizing. Among confirmed positions we count how many GRID levels are aligned with the TSMOM direction. Define

$$k_{i,t} = \begin{cases} k_{i,t}^{+} & \text{if } s_{i,t} = +1 \\ k_{i,t}^{-} & \text{if } s_{i,t} = -1 \\ 0 & \text{if } s_{i,t} = 0 \end{cases}$$

so $k_{i,t}$ is the number of thresholds crossed in the direction of the TSMOM signal. The GRID multiplier is then

$$m_{i,t} = \begin{cases} 0 & \text{if } c_{i,t} = 0 \\ 1 + \text{step\_up}\,(k_{i,t} - 1) & \text{if } c_{i,t} = 1 \text{ and } k_{i,t} \geq 1 \end{cases}$$

truncated to the interval $[0, \text{max\_mult}]$. A confirmed position that crosses only the first level receives multiplier 1 a position that crosses two or three levels receives a larger multiplier that increases linearly with $k_{i,t}$ up to max_mult.

These ingredients are combined with volatility targeting to form the final weights. Before enforcing the gross exposure cap the volatility scaled weight for contract i at time t is proportional to

$$\tilde{w}_{i,t} \propto m_{i,t}\, s_{i,t}\, \frac{\text{VOL\_TARGET}}{\hat{\sigma}_{i,t}}.$$

The factor $s_{i,t}$ sets the direction the factor $\text{VOL\_TARGET}/\hat{\sigma}_{i,t}$ delivers the usual inverse volatility scaling and the GRID multiplier $m_{i,t}$ amplifies or attenuates that base position depending on the strength and confirmation of the raw trend. Finally the vector of pre cap weights $\tilde{w}_t$ is rescaled so that the sum of absolute weights equals the chosen gross_cap. This produces a portfolio that preserves the basic structure of the volatility targeted TSMOM strategy but deploys its risk budget in a way that explicitly reflects both the presence and the strength of underlying trends.

6.3 Integrated Grid Search Setup

The integrated grid search jointly explores

- **Volatility lookback** (EWMA halflife in months)

  $$\text{vol\_halflife} \in \{3, 6, 9, 12\}.$$

- **Momentum signal lookback** (vol scaled returns in months)

  $$\text{sig\_lookback} \in \{6, 9, 12\}.$$

- **GRID lookback** (raw returns in months)

  $$\text{grid\_lookback} \in \{3, 6, 9, 12\}.$$

- **GRID levels** as strictly increasing triplets

  $$(\ell_1, \ell_2, \ell_3) \in \text{GRID\_LEVELS},$$

  where each component is drawn from

  $$\{0.01, 0.02, 0.03, 0.04\},$$

  subject to $0.01 \leq \ell_1 < \ell_2 < \ell_3 \leq 0.04$.

- **STEP_UP** as the incremental increase in size per additional level crossed

  $$\text{step\_up} \in \{0.25, 0.50, 0.75\}.$$

- **MAX_MULT** as the cap on the maximum position multiplier

$$\text{max\_mult} \in \{1.25, 1.50, 2.00\}.$$

- **GROSS_CAP** as the cap on portfolio gross exposure measured on volatility scaled weights

$$\text{gross\_cap} \in \{2, 4, 6\}.$$

For each parameter tuple the procedure is

1. Compute EWMA volatilities with the chosen `vol_halflife` and scale instrument returns to the common volatility target VOL_TARGET.
2. Build TSMOM signals using `sig_lookback` months of vol scaled returns and construct baseline EQ–Active weights that equal weight all nonzero signals.
3. Compute the GRID signal from raw returns using `grid_lookback` months of history counting how many of the thresholds (ℓ_1, ℓ_2, ℓ_3) are exceeded in the positive and negative directions.
4. Apply a confirmation filter that keeps only positions where the TSMOM sign agrees with the GRID direction and sets all other weights to zero.
5. Map the number of aligned levels into a multiplier $m_{i,t} \in [0, \text{max\_mult}]$ using `step_up` multiply the baseline EQ–Active weights by $m_{i,t}$ and rescale if necessary to satisfy the gross exposure cap `gross_cap`.

In summary the final volatility scaled portfolio weights satisfy

$$w_{i,t} \propto m_{i,t} \cdot \frac{s_{i,t} \cdot \text{VOL\_TARGET}}{\hat{\sigma}_{i,t}},$$

where $s_{i,t} \in \{-1, 0, +1\}$ is the TSMOM direction $\hat{\sigma}_{i,t}$ is the EWMA volatility estimate and $m_{i,t}$ encodes the GRID strength and confirmation. The Cartesian product of these sets yields a total of

$$4 \times 3 \times 4 \times 4 \times 3 \times 3 \times 3 = 5{,}184$$

distinct configurations evaluated in the grid search.

6.4 Transaction Costs

All strategies are evaluated net of trading frictions using

- A trading cost proportional to portfolio turnover equal to 1.5 bps per unit of turnover.
- A rolling cost applied each month based on gross exposure equal to 0.5 bps per month.

Net returns are obtained after subtracting both trading and rolling costs and the grid search ranks configurations by net Sharpe ratio.

6.5 In Sample Performance (1969-01-31 to 2014-12-31)

The best configuration by net Sharpe over the IS period is

$$\begin{aligned} \text{vol\_halflife} = 3, \quad \text{sig\_lookback} &= 9, \quad \text{grid\_lookback} = 9, \\ \text{grid\_levels} = (0.01, 0.02, 0.03), \quad \text{step\_up} &= 0.75, \\ \text{max\_mult} = 1.25, \quad \text{gross\_cap} &= 6.0, \quad \text{VOL\_TARGET} = 0.40. \end{aligned}$$

At these best parameters we compare three variants

1. **Baseline (no GRID)** standard vol scaled TSMOM (EWMA) with equal weighting across active instruments.
2. **TSMOM + GRID (Unsized + Confirmed)** apply the GRID confirmation filter but keep exposure at the baseline level with no strength based sizing.
3. **TSMOM + GRID (Sized by Levels)** full GRID confirmation and level based sizing using the multipliers $m_{i,t}$ described above.

The corresponding net performance statistics annualized and after all costs are

- **Baseline (no GRID)** net return 13.58% volatility 15.57% Sharpe 0.87.
- **GRID Unsized (Confirmed)** net return 15.22% volatility 16.26% Sharpe 0.94.
- **GRID Sized by Levels** net return 17.01% volatility 19.06% Sharpe 0.89.

Average monthly trading and rolling costs for the sized variant are approximately 0.44 bps and 0.51 bps. All three strategies are active in about 91.8% of months.

Table 5 summarizes the main net performance statistics while Table 6 reports selected microstructure measures. Leverage and imbalance are computed from the volatility scaled weights $w_{i,t}$ that feed into the gross exposure plots whereas turnover is based on the PnL weights used in the return calculation. We measure dollar imbalance as the net dollar exposure

$$\text{dollar\_strat}_t = \sum_i w_{i,t},$$

so positive values correspond to a net long book and negative values to a net short book.

Table 5: Baseline vs GRID strategies at best parameters (IS net of costs).

Statistic	Baseline EQ–Active	GRID Unsized	GRID Sized (Best)
AnnMean (net)	13.58%	15.22%	17.01%
AnnVol (net)	15.57%	16.26%	19.06%
AnnSharpe (net)	0.87	0.94	0.89

Figure 8 shows that both GRID variants improve on the baseline EQ–Active TSMOM with the sized by levels strategy delivering the highest terminal wealth over the IS window. Figure 9 illustrates how the GRID sizing mechanism affects the vol scaled gross exposure of the portfolio and how often the strategies operate near the leverage cap of six.

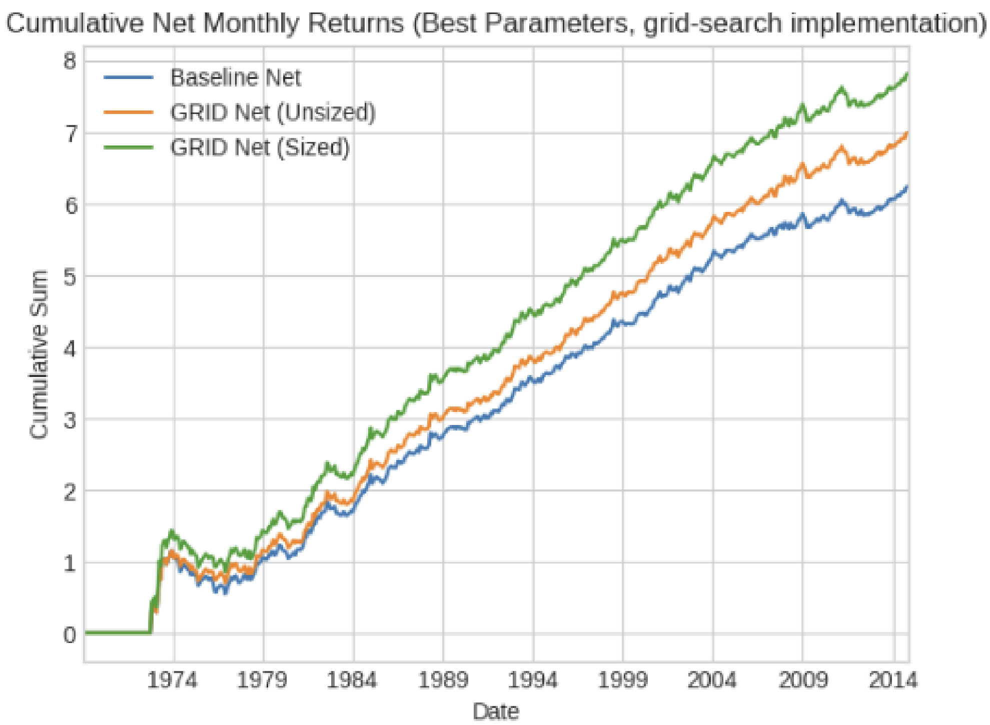

Figure 8: In sample cumulative net monthly returns for the baseline EQ–Active strategy the GRID Unsized confirmation strategy and the GRID Sized strategy at the best parameter configuration (1969-01-31 to 2014-12-31).

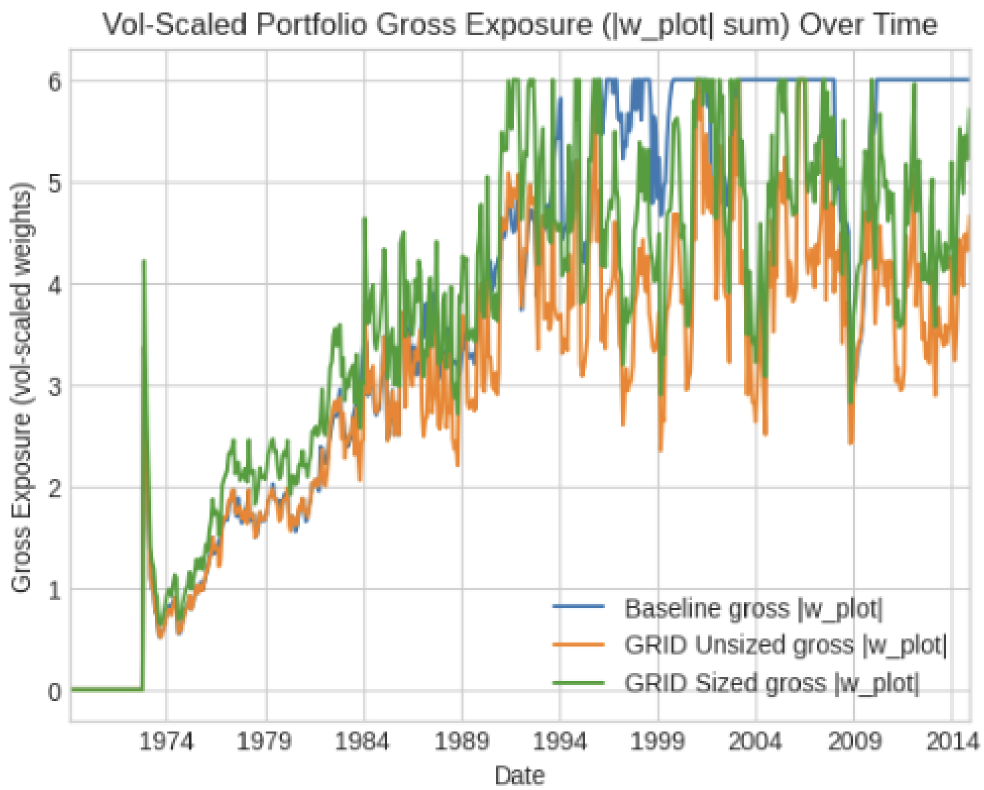

Figure 9: In sample vol scaled portfolio gross exposure measured as the sum of absolute volatility scaled weights for the same three strategies.

Table 6: Selected leverage turnover and net dollar imbalance statistics (IS).

Statistic	Baseline	GRID Unsized	GRID Sized (Best)
Avg leverage	3.777	2.996	3.595
95th pct leverage	6.000	5.064	6.000
Avg monthly turnover (PnL)	0.241	0.303	0.292
Approx annual turnover (PnL)	2.90	3.64	3.51
Avg inbalance (sum w)	0.924	0.758	0.898
5th pct inbalance	-2.803	-1.785	-2.111
95th pct inbalance	5.107	3.496	4.096
Avg # of positions	33.3	29.5	29.5

6.6 Qualitative Behavior

The GRID sized strategy

- Increases exposure when trends in raw returns are strong and aligned with the TSMOM direction so that multiple GRID levels are crossed.
- Reduces exposure toward zero when trends are weak noisy or not confirmed by the GRID signal.
- Operates close to the vol scaled gross exposure cap for a substantial fraction of the sample which explains the higher average leverage and somewhat higher turnover relative to the baseline and GRID Unsized variants.
- Maintains a diversified cross section with roughly thirty active instruments on average and exhibits a modest net long tilt as reflected in the positive average net dollar exposures.

6.7 Out of Sample Test Results

To assess robustness the best in sample configuration is frozen and applied to the full sample. The data span 1969-02-28 to 2025-05-30 and we define an out of sample (OOS) window from 2015-01-01 onward. The net series for all three strategies runs from 1969-02-28 to 2025-05-30 with the first OOS observation on 2015-01-30 and the last on 2025-05-30.

Using the fixed parameters we evaluate the same three variants in the OOS window. The baseline EQ–Active strategy delivers a net annualized return of about 4.28% on volatility of 12.83% for a Sharpe ratio of 0.33. The GRID Unsized specification which applies the confirmation filter but keeps baseline position sizes improves the net annualized return to 4.98% with volatility of 14.18% and a Sharpe ratio of 0.35. The GRID Sized strategy which additionally scales exposure with GRID strength attains a net annualized return of 5.14% with volatility of 15.17% and a Sharpe ratio of 0.34. The cumulative return plot shows that all three strategies track one another closely with similar drawdowns around 2020 and the GRID variants finishing slightly ahead of the baseline.

Risk usage differs more clearly across the three OOS strategies. The baseline portfolio has average vol scaled gross exposure of 5.36 with a ninety fifth percentile at the cap of 6.0. The GRID Unsized portfolio uses less leverage on average with vol scaled gross exposure around 3.81 and a ninety fifth percentile of 5.22 while the GRID Sized portfolio sits in between with average gross exposure of 4.53 and a ninety fifth percentile at 6.0. Turnover again increases from baseline to GRID Unsized to GRID Sized with average monthly values of 0.300 0.392 and 0.370 which implies annualized turnovers near 3.61 4.71 and 4.44. Trading and rolling costs are highest for the sized model yet remain below one basis point per month for all three specifications.

Out of sample dollar imbalance continues to show a modest net long tilt. The average net dollar value is 1.130 for the baseline 0.276 for GRID Unsized and 0.336 for GRID Sized with fifth and ninety fifth percentiles that span both net short and net long regimes. The average number of active contracts is about 56 for the baseline and roughly 48 for the GRID variants so the overlay slightly reduces cross sectional breadth while concentrating risk in markets where the trend is both strong and confirmed.

Table 7: OOS performance comparison from 2015-01-01 to 2025-05-30 net of costs.

Statistic	Baseline	GRID Unsized	GRID Sized
AnnMean (net)	4.28%	4.98%	5.14%
AnnVol (net)	12.83%	14.18%	15.17%
AnnSharpe (net)	0.33	0.35	0.34
Avg leverage	5.360	3.806	4.532
Avg monthly turnover	0.300	0.392	0.370
Approx annual turnover	3.61	4.71	4.44
Avg inbalance	1.130	0.276	0.336
Avg # of positions	56.1	47.6	47.6

7 Discussion

The updated experiments confirm that a GRID overlay can add value to a volatility targeted time series momentum benchmark yet the way that value is delivered depends on how aggressively the strategy uses the vol scaled leverage budget.

In sample with a volatility target of forty percent and a gross cap of six the baseline EQ–Active strategy already produces strong performance. The net annualized return is about 13.6% with volatility near 15.6% which implies a Sharpe ratio of 0.87. The GRID Unsized variant improves the net return to roughly 15.2% and raises the Sharpe ratio to 0.94 while actually using less vol scaled leverage on average. Its mean gross exposure drops to about 3.0 from 3.8 in the baseline and the ninety fifth percentile falls below the cap to roughly 5.1. This reflects the confirmation filter that turns off positions when the raw return signal disagrees with the TSMOM direction so risk is concentrated only in trends that look cleaner across both measures.

The GRID Sized strategy pushes the idea one step further by scaling exposure with the number of GRID levels that are aligned with the signal. In sample it delivers the highest net annualized return at about 17.0% but with higher volatility around 19.1% so its Sharpe ratio at 0.89 sits between the baseline and GRID Unsized. Average vol scaled leverage for the sized strategy is about 3.6 with a ninety fifth percentile again at the cap of six. Relative to the baseline the sized model gives up some Sharpe in exchange for a higher mean return and more time spent near the leverage ceiling.

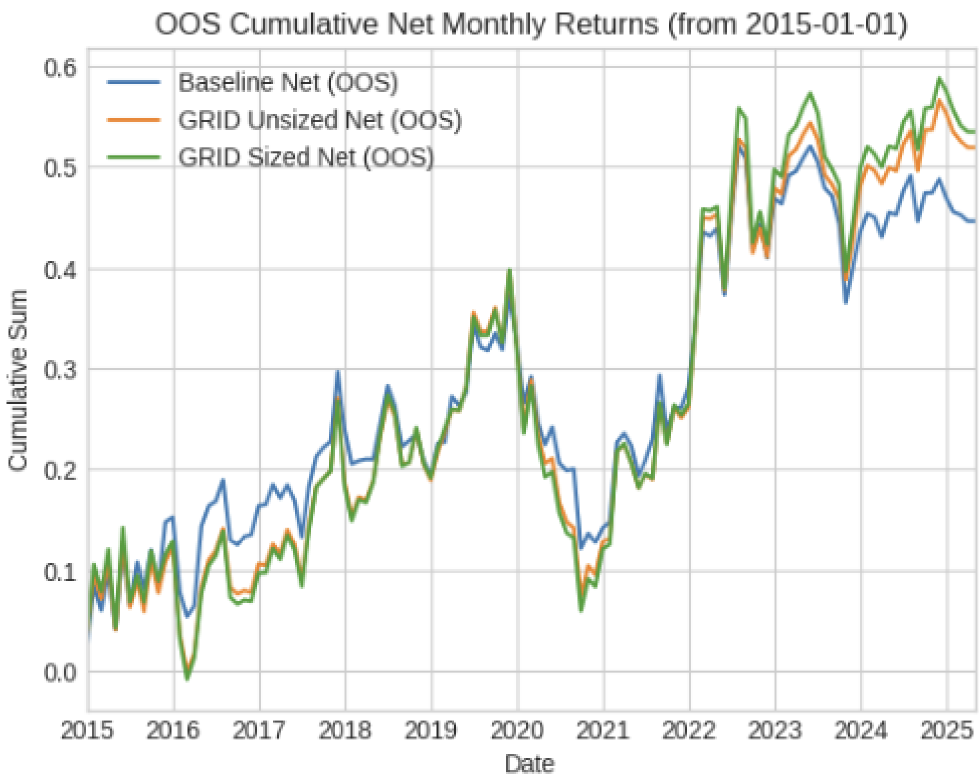

Figure 10: OOS cumulative net monthly returns for the baseline EQ–Active strategy the GRID Unsized strategy and the GRID Sized strategy using the best in sample parameters and starting from 2015-01-01.

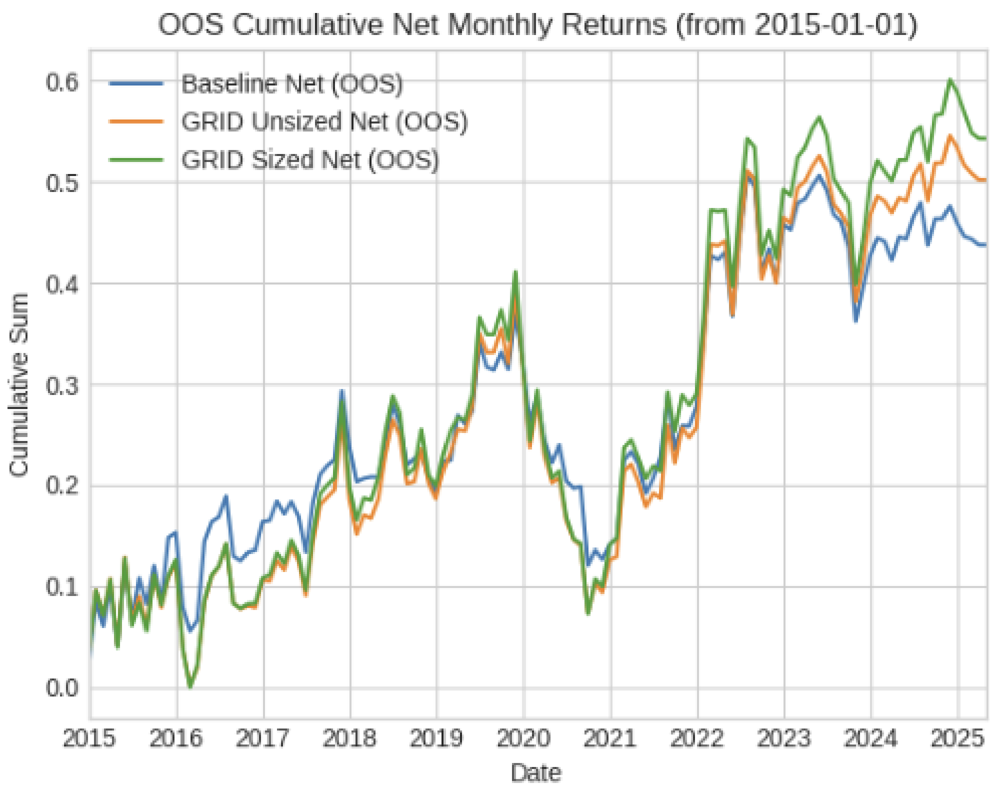

Figure 11: OOS vol scaled portfolio gross exposure measured as the sum of absolute volatility scaled weights for the baseline EQ–Active strategy the GRID Unsized strategy and the GRID Sized strategy.

Turnover numbers for all three strategies remain in a relatively tight band between roughly 2.9 and 3.6 turns per year which shows that the GRID overlay shifts where and when risk is used rather than simply trading much more.

The net dollar statistics clarify how directional the portfolios are when expressed in vol scaled terms. In sample the baseline has an average net dollar exposure around 0.92 while the GRID Unsized and GRID Sized variants show slightly smaller values of about 0.76 and 0.90. The tails extend to both negative and positive values which indicates that the portfolios continue to rotate between net long and net short regimes over time. At the same time the GRID variants operate with fewer contracts on average about thirty compared with thirty three for the baseline so the overlay concentrates exposure in a smaller but still diversified cross section.

Out of sample from 2015 onward the ranking across specifications remains consistent with the in sample picture but the differences narrow. The baseline EQ–Active strategy earns a net annualized return of 4.28% with volatility of 12.83% for a Sharpe ratio of 0.33. The GRID Unsized model delivers about 4.98% at 14.18% volatility and a Sharpe ratio of 0.35. The GRID Sized model produces 5.14% at 15.17% volatility and a Sharpe ratio of 0.34. The cumulative net return curves show that the three strategies track each other closely with very similar drawdowns around 2020 and only a modest spread in terminal wealth by mid 2025.

Risk usage in the out of sample window highlights an important difference between the designs. The baseline runs with the highest average vol scaled gross exposure about 5.36 and frequently touches the cap of six. The GRID Unsized strategy is noticeably more conservative with an average of 3.81 and a ninety fifth percentile near 5.2 while the GRID Sized strategy sits between the two with an average of 4.53 and a ninety fifth percentile at the cap. Turnover is highest for the GRID Unsized version at about 4.7 turns per year while the baseline and GRID Sized strategies are slightly lower near 3.6 and 4.4. Average trading and rolling costs remain comfortably below one basis point per month across all variants so they do not dominate performance.

Dollar imbalance out of sample also changes across specifications. The baseline shows the strongest net long tilt with an average net dollar value around 1.13 in vol scaled units and tails that reach beyond plus or minus three. The GRID overlays reduce this tilt substantially. Average net dollar is roughly 0.28 for GRID Unsized and 0.34 for GRID Sized and the range between the fifth and ninety fifth percentiles tightens as well. At the same time the number of active contracts drops from about fifty six in the baseline to roughly forty eight in both GRID variants which indicates that the overlay becomes more selective across markets when volatility scaled capacity is limited.

Taken together these results suggest that the main role of the GRID confirmation and sizing rules under a high volatility target and a generous gross cap is to improve the allocation of risk rather than to expand the risk budget. In sample the confirmation step boosts Sharpe by cutting back on noisier trends while the sizing rule converts the remaining high quality signals into higher mean returns at the cost of higher volatility. Out of sample the same structure continues to deliver incremental gains over the baseline though the edge is modest and the absolute performance is more sensitive to market regimes and to how aggressively the vol scaled leverage cap is used.

8 Conclusion and Future Work

This study shows that a GRID based overlay can enhance a volatility targeted time series momentum strategy in a diversified futures universe when returns are scaled to a high volatility target and a generous vol scaled gross cap. In sample the baseline EQ–Active strategy delivers a solid Sharpe ratio of 0.87 with a net annualized return of about 13.6%. Adding GRID as a pure confirmation filter increases the Sharpe ratio to 0.94 and raises the net return to roughly 15.2% while using less vol scaled leverage on average. Extending the overlay to scale positions by GRID strength produces the highest mean return at about 17.0% with a Sharpe ratio of 0.89 and somewhat higher volatility and leverage. These findings indicate that conditioning on trend quality and strength can improve how a fixed risk budget is deployed across markets and across time.

Out of sample from 2015 to 2025 the same parameter set continues to add value though with much smaller margins. All three strategies remain profitable after costs yet their Sharpe ratios cluster between 0.33 and 0.35. The GRID Unsized and GRID Sized variants modestly outperform the baseline in terms of net return while operating with lower or intermediate vol scaled leverage and a more moderate net long tilt. This pattern suggests that the GRID overlay mainly improves the allocation of risk rather than changing the overall level of risk taking and that its edge is sensitive to the prevailing volatility and trend environment.

Several extensions follow naturally from these results. First future work can replace the single in sample grid search with rolling or cross validated procedures that explicitly target robustness and quantify parameter uncertainty. Second the volatility target the vol scaled gross cap and the GRID thresholds can be made regime dependent using simple indicators such as realized volatility or macro variables so that the strategy does not operate near the leverage ceiling in all conditions. Third richer execution models that incorporate market depth and contract specific liquidity would allow a more realistic assessment of capacity especially for the higher turnover GRID variants. Finally the GRID mechanism can be combined with cross sectional momentum carry or value signals in order to study whether trend strength based sizing improves multi style allocation in large futures portfolios.

Market	**Macro**	**Description**
Bear Market (0)		
0	0	Strong mean reversion; macro still solid.
0	1	Mixed; improving macro buffers downside.
0	2	Best regime; high forward returns.
0	3	Crisis environment; negative returns.
Neutral Market (1)		
1	0	Mild positive outcomes.
1	1	Constructive, mid-range returns.
1	2	Macro strong; market undecided.
1	3	Weak; recession dominates.
Bull Market (2)		
2	0	Strong pro-risk environment.
2	1	Robust; improving macro + momentum.
2	2	Classic risk-on; positive and stable.
2	3	Worst regime; bull traps in recession.

Table 8: Twelve combined macro–market regimes with concise characterizations.

References

[Moskowitz et al., 2012] Moskowitz, T. J., Ooi, Y. H., and Pedersen, L. H. (2012). Time series momentum. *Journal of Financial Economics*, 104(2):228–250.